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1. Introduction

The heat transfer model can be used to predict the engine heat transfer rate, which is very important for thermal load analysis, combustion performance prediction, and cycle simulation. Most investigations of heat transfer in spark ignition (S.I.) engines are large-scale ones. Oguri [\[1\]](#page-11-0) used Eichelberg's model to predict the heat transfer rate of a 1400 c.c. S.I. engine, yielding predicted results that agreed with the experimental results for the expansion stroke, but not for the compression stroke. This might be because the model was established for diesel engines and was not suitable for S.I. engines. Alkidas [\[2,3\]](#page-11-0) measured the instantaneous heat flux on the cylinder head of an 820 c.c. four-stroke S.I. engine and found that it could be affected by the engine speed, air–fuel ratio, and volumetric efficiency. Alkidas and Suh [\[4\]](#page-11-0) investigated the effects of swirl or tumble motion on the heat transfer and combustion characteristics of a single-cylinder four-valve 400 c.c. S.I. engine. It was found that increasing swirl or tumble motions might raise the peak rate of heat release, the local surface temperature, and heat flux, on the cylinder head. Shayler et al. [\[5\]](#page-11-0) utilized two methods to obtain the instantaneous heat flux of the combustion chamber. In the first method, the first law of thermodynamics was applied to calculate the heat transfer rate, but the results were not accurate due to uncertain gas properties. In the second method, the heat transfer rates were calculated based on Woschni's, Annand's, and Eichelberg's experimental models. It was found that Eichelberg's model could produce prediction results closest to the experimental data.

ABSTRACT

A heat transfer model for small-scale spark-ignition engines has been proposed by authors in previous study. However, that model was developed based on experimental data of one engine, it may not be so accurate for others. In order to improve the accuracy of predicted heat transfer rate for different engines, a modified heat transfer model using Stanton number based on two engines is proposed. Prediction results of instantaneous heat flux, global heat transfer, heat release rate, and cylinder pressure based on the proposed model are compared with the experimental results of three engines and prediction results of previous model. It is found that the proposed model has prediction results closer to the measured data than the previous models at the most engine operation conditions.

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Previous studies have focused on developing heat transfer models for large-scale engines. However, the heat transfer characteristics of large-scale and small-scale engines are quite different. For large-scale S.I. engines, about one-third of the fuel energy is transformed to heat loss from the cylinder body. But for a small-scale 125 c.c. two-stroke S.I. engine, Franco and Martorano [\[6\]](#page-11-0) found that approximately 50% of the fuel energy is converted to heat loss, which is much higher than that of large-scale S.I. engines. Since the heat transfer characteristics of small-scale engine are different from that of large-scale engine, the authors of this paper [\[7\]](#page-11-0) had ever proposed a heat transfer model specific for four-stroke small-scale S.I. engines using Stanton number. However, that model was developed based on experimental data of one engine, it may not be so accurate for others.

In order to improve the accuracy of the predicted heat transfer rate for different scooter engines, a heat transfer model also using the Stanton number is proposed and based on two 125 c.c. aircooled four-stroke S.I. engines. The remainder of this paper is organized as follows. Section 2 introduces previous heat transfer model. The heat flux obtained from the measured temperatures is then used to establish the heat transfer model in Section [3](#page-1-0). The heat transfer model is used for engine simulation and the engine model is described in Section [4](#page-6-0). Simulation results of cylinder pressure are compared with the experimental results of three engines in Section [5](#page-7-0). Finally, conclusions are made in Section [6.](#page-9-0)

2. Heat transfer model

The heat transfer rate \dot{Q}_{ht} from the flowing gas to the combustion chamber wall is dominated by the forced convection [\[7,8\]](#page-11-0) and can be expressed as:

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Nomenclature

$$
\frac{dQ_{ht}}{dt} = hA(T_g - T_{sp})
$$
\n(1)

where A is the area of the heat transfer surface, which is two times the piston area, i.e. $2A_p$, for obtaining global heat transfer rate [\[7\]](#page-11-0); h is the empirical heat transfer coefficients, which is assumed the same for the entire heat transfer surface $[8-11]$; T_{sp} is the measured temperature of the spark plug temperature; and T_g is the temperature of the flowing gas, which can be obtained using the state equation of ideal gas [\[7,9,12\]](#page-11-0) as follows:

$$
T_g = \frac{P_{cyl} V_{cyl}}{m_a R} \tag{2}
$$

where P_{cyl} is the cylinder gas pressure, V_{cvl} is the cylinder volume, m_a is the air mass in the cylinder, and R is the gas constant.

In order to provide a simple heat transfer model, but yet accurate enough, for the engine simulation, a heat transfer coefficient using the Stanton number S_t is proposed in this paper, as follows:

$$
h = S_t \rho c_p u \tag{3}
$$

where u is the gas turbulent fluctuating velocity, which can be approximated as $0.5C_m$ under the assumption of open combustion chamber without swirl [\[9\].](#page-11-0) The factor c_p is the specific heat at constant pressure, and can be expressed as:

$$
c_p = \frac{R}{1 - (1/\gamma)}
$$
\n⁽⁴⁾

where γ is the specific heat ratio, which can be obtained using the following equation [\[13\]](#page-11-0):

$$
\gamma = 1.338 - 6 \times 10^{-5} T_g + 10^{-8} T_g^2 \tag{5}
$$

The Stanton number S_t , which is proposed by previous study of Wu et al. [\[7\]](#page-11-0) based on Suzuki AN125 scooter engine (as Table 1), is determined as a function of the mean piston speed C_m and is expressed as

$$
S_t = 0.718 \exp(-0.145 C_m)
$$
 (6)

Table 1

Since Eq. (6) was proposed based on single engine, it may not be so accurate for other engines. Another heat transfer model also using Stanton number is developed in this paper.

3. Heat flux analysis

3.1. Experimental setup

Two 125 c.c. four-stroke air-cooled spark-ignition engines with single cylinder were employed for developing the heat transfer coefficient in this paper, with specifications as shown in Table 1. Moreover, the combustion chambers of both engines are hemisphere-shaped, as shown in [Fig. 1](#page-2-0).

Five thermocouples were used to measure the temperatures of the cylinder head at specific positions, as shown in [Fig. 2](#page-2-0) and [Table](#page-2-0) [2](#page-2-0). Two E-type coaxial thermocouples (Medtherm TCS-102-E), th_1 and $th₂$, were used to measure the instantaneous temperatures of the inside surface of the cylinder wall near the exhaust and intake valves, respectively. The E-type thermocouples offer measurement ranges from -270 to $+1000$ °C and temperature tolerance of

Fig. 1. Combustion chambers of both engines.

Fig. 2. Inside view of the locations of thermocouples.

Table 2 Detail location of thermocouples.

Note: D1 is the distance between the th_{sp} and the spark plug. D2 is the distance from the $th_{2,4}$ to the spark plug. D3 is the distance between the spark plug and cylinder wall. D4 is the distance from the $th_{1,3}$ to the spark plug.

 \pm 1.7 °C or 0.75% [\[14\].](#page-11-0) Two K-type thermocouples with Omega 650 temperature indicator, th₃ and th₄, were used to measure the steady-state temperatures of the outside surface of the cylinder wall near the exhaust and intake valves, respectively. Another K-type thermocouple, th_{sp} , was used to measure the temperature of the spark plug T_{sp} . The K-type thermocouples offer measurement ranges from -270 to +1372 C and temperature tolerance of ± 2.2 °C or 0.75% [\[14\].](#page-11-0) Moreover, since the measuring probe of Etype and K-type thermocouples are very small, the sizes are 1/32 and 1/16 inch, respectively, which can minimize the error of measured temperature.

The mean cylinder gas temperature T_g was estimated using the state equation of ideal gas as shown in Eq. [\(2\)](#page-1-0) with the cylinder pressure, which was measured with the spark-plug type pressure transducer (Kistler 6117A37).

In order to obtain the engine heat transfer rate for a wide-range of operating conditions, the engine speed was adjusted from 3000 to 6000 rpm at increments of 1000 rpm, and the brake mean effective pressure (BMEP) used as the engine load was adjusted from 1 to 7 bar with an increment of 1 bar.

3.2. Heat flux calculation

In most heat flux calculation, the heat flux through the cylinder head wall is assumed to be one-dimensional unsteady heat conduction [\[1,4,9–11\]](#page-11-0). Since the unsteady heat conduction of the inwall temperature field exists only within a very small distance from the wall surface, the unsteady component of the temperature gradient perpendicular to the surface is usually much larger than that parallel to the surface. Therefore, one-dimensionality is safely assumed for the unsteady component of the surface heat flux calculation [\[10\].](#page-11-0) The heat flux at the combustion chamber can be obtained by solving the following partial differential equation with two boundary conditions:

$$
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial^2 x} \tag{7}
$$

where T is the temperature of the cylinder wall, which is a function of t and x; t is the time; x is the distance from the wall surface; $\alpha = k/2$ ρc is the thermal diffusivity; k is the thermal conductivity; and c is the specific heat. The boundary conditions are defined as follows:

$$
T(0,t) = T_{wi}(t)
$$
 at $x = 0$
\n
$$
T(\ell, t) = T_{wo}(t) = \text{constant at } x = \ell
$$
 (8)

where T_{wi} is the instantaneous temperature of the cylinder inside wall surface and T_{wo} is the steady-state temperature of the cylinder outside wall surface with a distance ℓ from the inside wall surface. The material properties of the cylinder head with k and α are functions of temperature [\[15\],](#page-11-0) as shown in Fig. 3.

First, T_{wi} is represented by the following Fourier series [\[16\]:](#page-11-0)

$$
T_{wi} = T_{wm} + \sum_{n=1}^{N} [A_n \cos(n\omega t) + B_n \sin(n\omega t)] \tag{9}
$$

where T_{wm} is the time-averaged value of T_{wi} ; ω is the angular frequency of temperature variation, which is one half of the engine

Fig. 3. Material thermal properties [\[15\].](#page-11-0)

angular velocity for the four-stroke engine; and N is the harmonic number which is set to be 200 in this paper for obtaining more accurate representation without more computation time [\[7\].](#page-11-0)

The solution of Eq. [\(7\)](#page-2-0) can be expressed as

$$
T(x,t) = T_{wm} - (T_{wm} - T_{wo}) + \sum_{n=1}^{\infty} e^{-c_n x} F_n(x,t)
$$
 (10)

where

$$
F_n = A_n \cos(n\omega t - c_n x) + B_n \sin(n\omega t - c_n x)
$$
\n(11)

$$
A_n = \frac{2}{\tau} \int_0^{\tau} T_w(t) \cos(\omega nt) dt
$$
 (12)

$$
B_n = \frac{2}{\tau} \int_0^{\tau} T_w(t) \sin(\omega nt) dt
$$
 (13)

$$
c_n = \sqrt{\frac{n\omega}{2\alpha}}\tag{14}
$$

The instantaneous heat flux at the cylinder inside wall surface, i.e. $x = 0$, can then be obtained using Fourier's law, as follows:

$$
q_w(t) = -k \left(\frac{\partial T}{\partial x}\right)_{x=0}
$$

= $\frac{k}{\ell} (T_{wm} - T_{wo}) + k \sum_{n=1}^{N} c_n [(A_n + B_n) \cos(n\omega t) + (B_n - A_n) \sin(n\omega t)]$ (15)

Since the major heat flux is produced within compression and expansion strokes, q_w is calculated only for these two strokes. The measured temperatures of the cylinder inside wall surface for AN125 and CB125 at 6000 rpm with 7 bar BMEP are shown in Figs. 4 and 5, respectively.

As can be seen from Figs. 4 and 5, the temperature near the exhaust valve $T_{w_i,ex}$ is higher than that near the intake valve $T_{w_i,in}$, because the exhaust valve region is heated by the high-temperature exhaust. Both $T_{wi,ex}$ and $T_{wi,in}$ have large temperature gradients near TDC due to the combustion. Since the compression ratio of AN125 is larger than that of the CB125, which results in higher combustion temperature, as well as cylinder inside wall temperature.

The corresponding instantaneous heat fluxes $q_{w,ex}$ and $q_{w,in}$ of AN125 and CB125 can then be calculated for the exhaust and intake valve regions, respectively, as shown in Figs. 6 and 7. The greater the temperature gradient, the higher the heat flux is.

Fig. 4. Temperature variation of the cylinder inside wall surface for AN125.

Fig. 5. Temperature variation of the cylinder inside wall surface for CB125.

Fig. 6. Instantaneous heat flux variation of the cylinder inside wall surface for AN125.

Fig. 7. Instantaneous heat flux variation of the cylinder inside wall surface for CB125.

Two instantaneous heat fluxes at the exhaust and intake valve regions are used to represent the average instantaneous heat flux

Fig. 8. Heat flux with various engine loads at 6000 rpm for AN125.

Fig. 9. Heat flux with various engine loads at 6000 rpm for CB125.

 q_m , i.e. $q_m = (q_{w,in} + q_{w,ex})/2$. Figs. 8 and 9 show the experimental results of AN125 and CB125 with various load conditions at 6000 rpm. The instantaneous heat flux grows with increasing engine load, due to the gas heat transfer coefficient is increased with higher gas pressure and temperature [\[11,17\].](#page-11-0) Figs. 10 and 11 show the experimental results of AN125 and CB125 with different engine speeds at 6 bar BMEP. Since the gas pressure and temperature do not vary significantly with the engine speed under constant load, the gas heat transfer coefficient changes only slightly with various engine speeds, as does the instantaneous heat flux.

3.3. Curve fitted heat flux

For the engine simulation, the heat transfer rate can be obtained using Eq. [\(1\)](#page-1-0). In order to develop the proposed heat transfer model, the spark plug temperature is assumed to be T_{sp} in this paper. Since T_{sp} cannot be measured during simulation, the previous study [\[7\]](#page-11-0) proposed an experimental formula, which is defined as a function of engine speed and intake manifold pressure, to represent the response of T_{sp} . However, the previous proposed T_{sp} is developed based on single engine, i.e. AN125, it may not be suitable for other engines. Thus, another experimental formula is proposed for T_{sp} in this paper, and is determined as a function of intake manifold pressure P_{im} , engine speed ω_e , and compression ratio CR, as follows:

Fig. 10. Heat flux with various engine speeds at 6 bar BMEP for AN125.

Fig. 11. Heat flux with various engine speeds at 6 bar BMEP for CB125.

$$
T_{sp} = a_{sp1}\omega_e + a_{sp2}\omega_e^2 + a_{sp3}P_{im} + a_{sp4}\omega_eP_{im} + a_{sp5}P_{im}^2 + a_{sp6}CR
$$
\n(16)

where ω_e is the engine speed with the unit of rev/sec; P_{im} is the intake manifold pressure with the unit of bar; and a_{spl} is the coefficient obtained from the curve fitting of experimental results and is listed in Appendix A; The coefficient of determination R_S is used to evaluate correlations between the measured and predicted values of T_{sp} . R_S can be expressed as

$$
R_{S} = \frac{\sum_{i=1}^{n} (\hat{y}_{i} - \bar{y})^{2}}{\sum_{i=1}^{n} (y_{i} - \bar{y})^{2}}
$$
(17)

where y is the experimental data; \bar{y} is the mean value of experimental data; and \hat{v} is the predicted data. Here, R_s ranges between 0 and 1. If R_S is close to 1, the curve fitted result is close to actual value, and vice versa. The correlation results of AN125 and CB125 are shown in [Fig. 12](#page-5-0) and [Table 3](#page-5-0). Since R_S of the proposed model for the AN125 and CB125 are equal to 0.9667 and 0.9736, respectively, the predicted results are determined to be very close to the experimental data than the previous model.

In order to develop a heat transfer model for all operating conditions of two target engines, the Stanton number S_t is also employed for the proposed model, and can be obtained from the curve fitting of the experimental heat transfer coefficient using the following equation:

Fig. 12. Correlations between the measured and predicted T_{sp} for AN125 and CB125.

Table 3 R_S of predicted $T_{\rm sn}$.

	Previous	Proposed
AN125	0.9959	0.9983
CB125	0.1496	0.9965
Average	0.5727	0.9974

$$
h = \frac{q_m}{T_g - T_{sp}}\tag{18}
$$

As can be seen from Figs. 13 and 14, the Stanton number of the AN125 and CB125 are varied with engine speed, and presents an exponential tendency. Therefore, this paper employed an exponential function with the engine speed to represent the dynamics of Stanton number. Moreover, since the characteristic of heat transfer would be affected by engine geometry, such as compression ratio CR, stroke S, displaced cylinder volume $V_{\text{cyl}}(\theta)$. Thus the S_t can be expressed as follows:

$$
S_t = 5.91488CR \cdot S^2 \left[0.0089(10^5 V_{cyl}(\theta))^2 + \exp(-0.0106 C_m^2) \right]
$$
(19)

where the displaced cylinder volume $V_{\text{cyl}}(\theta)$ is a function of crank angle θ .

Fig. 13. Correlations of experimental Stanton number of AN125 and engine speed.

Fig. 14. Correlations of experimental Stanton number of CB125 and engine speed.

The correlations between the predicted and measured instantaneous heat flux for AN125 and CB125 are shown in Figs. 15 and 16, respectively. The corresponding results of R_S for AN125 and CB125 are shown in [Table 4](#page-6-0). Since the previous model is developed based on AN125, its prediction results are close to experimental data than proposed model. But the averaged R_S of the proposed model for both engines has larger R_s than previous one, i.e. the proposed model has more accurate prediction results than previous one about 30.3%.

Similar correlation results for the heat transfer at the cylinder head of AN125 and CB125 are shown in [Figs. 17 and 18](#page-6-0). The proposed model for both engines has closer prediction results than previous one, about 83.6%. As can be seen from [Table 5,](#page-6-0) since the R_S of proposed model for both engines are very close to 1, the predicted results of the proposed model are determined to be very close to experimental data.

In general, the heat release rate between the intake valve closed (IVC) and spark advance (SA) angle ($Q_{hr,IVC\rightarrow SA}$) should close to zero, due to that the combustion chamber is a closed system and the fuel energy is not released yet. Thus, this characteristic is employed to evaluate the proposed model. The heat release rate can be obtained using the experimental data of the cylinder pressure, and can be expressed as follows:

$$
\frac{dQ_{hr}}{d\theta} = \frac{c_v}{R} \left(p \frac{dV_{cyl}}{d\theta} + V_{cyl} \frac{dP_{cyl}}{d\theta} \right) + P_{cyl} \frac{dV_{cyl}}{d\theta} + \frac{dQ_{ht}}{d\theta}
$$
(20)

Fig. 15. Correlation between the predicted and measured instantaneous heat flux for AN125.

Fig. 16. Correlation between the predicted and measured instantaneous heat flux for CB125.

Table 4 R_s of instantaneous heat flux.

	Previous	Proposed
AN125	0.998	0.378
CB125	0.052	0.989
Average	0.525	0.684

Fig. 17. Correlation between the predicted and measured heat transfer at cylinder head for AN125.

Experimental data of cylinder pressure of AN125 and CB125 are used to estimate the heat release rate. The IVC and SA of AN125 are 325° and 270° , respectively. The calculated heat release rate of AN125 with the proposed model at 3000 rpm with 6 bar BMEP is shown in Fig. 19. As can be seen from Fig. 19, the $Q_{hr,IVC\rightarrow SA}$ of proposed model is close to zero, and its mean value of $\dot{Q}_{hr,IVC\rightarrow SA}$ is -0.287. Thus, the proposed heat transfer model has accurate estimation results between the IVC and SA.

For the CB125, the IVC and SA are 320° and 260° , respectively. The calculated heat release rate of CB125 with the proposed model at 3000 rpm with 6 bar BMEP is shown in [Fig. 20](#page-7-0). Similar to the previous calculated results, the $\dot{Q}_{hr,IVC\rightarrow SA}$ of the proposed model is also close to zero. Furthermore, the $\dot{Q}_{hr,IVC\rightarrow SA}$ of proposed model at 3000 rpm is -0.320. Thus, the proposed model also has accurate estimation results between the IVC and SA.

Fig. 18. Correlation between the predicted and measured heat transfer at cylinder head for CB125.

Fig. 19. The heat release rate of AN125 with the proposed model at 3000 rpm with 6 bar BMEP.

In order to evaluate the proposed model can be used for other operating conditions, the cylinder pressure of both engines are utilized to calculate heat release rate from 3000 to 6000 rpm with the BMEP from 2 to 7 bar. The calculated results of $\dot{Q}_{hr,IVC\rightarrow SA}$ are shown in [Table 6,](#page-7-0) which shows that the proposed model can be used for overall engine operating conditions, and its root mean square of $Q_{hr,IVC \rightarrow SA}$ is 0.403.

The results of $Q_{hr,IVC\rightarrow SA}$ at all operating conditions of CB125 are shown in [Table 7](#page-7-0), which also shows that the proposed model has accurate estimation results between the IVC and SA for overall engine operating conditions, and its root mean square of $\dot{Q}_{hr,IVC\rightarrow SA}$ is 0.502.

4. Engine model

An engine model, which is consisted of charging, combustion, and rotational dynamics, is established with Matlab/Simulink as shown in [Fig. 21](#page-7-0) [\[18,19\]](#page-11-0).

Fig. 20. The heat release rate of CB125 with the proposed model at 3000 rpm with 6 bar BMEP.

Table 6

Heat release rate $\dot{Q}_{hr,IVC\rightarrow SA}$ of AN125.

$\omega_{\rm e}$	BMEP						
	2 bar	3 bar	4 bar	5 bar	6 bar	7 bar	
3000 rpm 4000 rpm 5000 rpm 6000 rpm	-0.444 -0.257 -0.186 -0.99	-0.415 -0.219 -0.13 -0.65	-0.28 -0.165 -0.84 -0.28	-0.193 -0.124 -0.38 -0.15	-0.287 0.23 0.39 -0.29	0.35 0.107 0.115 0.57	

Note: Root mean square of $\dot{Q}_{hr,IVC \rightarrow SA} = 0.403$.

Table 7

Heat release rate $\dot{Q}_{hr,IVC\rightarrow SA}$ of CB125.

$\omega_{\rm e}$	BMEP						
	2 bar	3 bar	4 bar	5 bar	6 bar	7 bar	
3000 rpm	-0.132	-0.930	-0.940	-0.400	-0.320	0.161	
4000 rpm	-0.147	-0.108	-0.920	-0.670	0.494	0.860	
5000 rpm	-0.153	-0.135	-0.113	-0.610	-0.388	0.120	
6000 rpm	-0.151	-0.130	-0.117	-0.880	-0.360	-0.360	

Note: Root mean square of $\dot{Q}_{hr,IVC \rightarrow SA} = 0.502$.

The charge model is a filling-and-emptying model based on the isentropic compressible flow equation for predicting the air flow rate. It consists of non-choked and choked flow dynamics as shown in Eqs. (21) and (22), respectively.

$$
\dot{m}_{ai} = \frac{C_{d,im} A_{th} P_{atm}}{\sqrt{R_a T_{atm}}} \left(\frac{P_{im}}{P_{atm}}\right)^{\frac{1}{\gamma_a}} \left\{\frac{2\gamma_a}{\gamma_a - 1} \left[1 - \left(\frac{P_{im}}{P_{atm}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]\right\}^{\frac{1}{\gamma_a}},
$$
\n
$$
\text{when } \frac{P_{im}}{P_{atm}} > \left(\frac{2}{\gamma_a + 1}\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{21}
$$

$$
\dot{m}_{ai} = \frac{C_{d,im}A_{th}P_{atm}}{\sqrt{R_aT_{atm}}} \sqrt{\gamma_a} \left(\frac{2}{\gamma_a + 1}\right)^{\frac{\gamma_a+1}{2(\gamma_a - 1)}}, \text{when } \frac{P_{im}}{P_{atm}} \leq \left(\frac{2}{\gamma_a + 1}\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{22}
$$

where \dot{m}_{ai} is the mass airflow through the throttle body; $C_{d,im}$ is the discharge coefficient of the intake manifold; A_{th} is the cross-sectional area of the throttle body; R_a is the ideal gas constant of air; γ_a is the specific heat ratio of air; T_{atm} and P_{atm} are the temperature and pressure of the atmosphere, respectively. The intake manifold pressure P_{im} is obtained from the state equation of the ideal gas.

$$
\frac{dP_{im}}{dt} = \frac{R_{air}T_{im}}{V_{im}}(\dot{m}_{ai} - \dot{m}_{ao})
$$
\n(23)

where V_{im} is the volume of the intake manifold. \dot{m}_{ao} is the mass airflow from the intake manifold into the cylinder, which can be expressed as:

$$
\dot{m}_{ao} = \frac{P_{im} V_d}{R_{air} T_{im} \pi} \omega \eta_{\nu}
$$
\n(24)

where V_d is the displaced cylinder volume; ω is the engine speed; and η_v is the volumetric efficiency.

The combustion model can be represented using the heat release model, which is a zero-dimensional model based on the first law of thermodynamics [\[18,19\]](#page-11-0).

$$
\frac{dP_{cyl}}{d\theta} = \frac{\gamma_g - 1}{V_{cyl}} \left(\frac{dQ_{hr}}{d\theta} - \frac{dQ_{ht}}{d\theta} \right) - \frac{\gamma_g P_{cyl}}{V_{cyl}} \frac{dV_{cyl}}{d\theta}
$$
(25)

where γ_{g} is the specific heat ratio of cylinder gas; P_{cyl} and V_{cyl} are the pressure and volume of the cylinder, respectively; and θ is the crank angle. The heat transfer rate is calculated from Eqs. [\(1\), \(3\) and \(19\).](#page-1-0) The heat release rate with respect to the crank angle can be obtained from the rate of mass fraction burned as:

$$
\frac{dQ_{hr}}{d\theta} = a_1 \frac{a_2 + 1}{\theta_d} \left(\frac{\theta - \theta_0}{\theta_d}\right)^{a_2} \exp\left[-a_1 \left(\frac{\theta - \theta_0}{\theta_d}\right)^{a_2 + 1}\right] Q_{HV} m_f \tag{26}
$$

where m_f is fuel mass injected into the cylinder; θ_d is the total combustion duration expressed in crank angle; θ_0 is the start of combustion; Q_{HV} is the heating value of fuel; a_1 and a_2 are 5 and 2, respectively [\[9\].](#page-11-0)

5. Verification results

Experimental data of cylinder pressure are used to verify the proposed engine model. The comparison of cylinder pressures for AN125 at 3000 and 6000 rpm with 6 bar BMEP (full load) are shown in [Figs. 22 and 23,](#page-8-0) respectively. Although the proposed model has larger prediction error than previous one, its maximum prediction errors are about 9% and 5% for 3000 and 6000 rpm, respectively. As can be seen from [Figs. 15 and 17,](#page-5-0) the proposed model underestimates the heat flux at the cylinder head, which results in overestimated cylinder pressure. Since the previous model

Fig. 21. Block diagram of the engine model.

Fig. 22. Comparison of the cylinder pressure at 3000 rpm with 6 bar BMEP for AN125.

Fig. 23. Comparison of the cylinder pressure at 6000 rpm with 6 bar BMEP for AN125.

is developed based on AN125, its prediction results is closer to the experimental data than proposed one.

The comparison of cylinder pressures for CB125 at 3000 and 6000 rpm with 6 bar BMEP are shown in Figs. 24 and 25, respec-

Fig. 24. Comparison of the cylinder pressure at 3000 rpm with 6 bar BMEP for CB125.

Fig. 25. Comparison of the cylinder pressure at 6000 rpm with 6 bar BMEP for CB125.

tively. Both figures show that the proposed model has the predicted results closest to the experimental data than previous one. As can been seen from [Figs. 16 and 18,](#page-6-0) because the previous model overestimates the heat flux at the cylinder head significantly, which results in underestimates cylinder pressure.

In order to verify the proposed model can also be used for the other engines, another experimental data of cylinder pressure (Yamaha XC125) is conducted in this paper. The specification of the XC125 is shown in Table 8.

Since the heat transfer rate is obtained based on the difference of temperature (see Eq. [\(1\)\)](#page-1-0), its estimation accuracy is significantly effected by the predicted T_{sp} . In order to verify the proposed T_{sp} empirical formula, the experimental data of cylinder pressure (Yamaha XC125) is utilized. The correlation results of the previous empirical formula [\[7\]](#page-11-0) and proposed one for XC125 are shown in [Fig. 26.](#page-9-0) The previous formula underestimates the T_{sp} . The R_S of the previous formula and proposed one are 0.3451 and 0.9567, respectively. Thus, the predicted results of the proposed formula are determined to be very close to the experimental data than the previous one.

Experimental data of cylinder pressure of XC125 are also used to estimate the heat release rate, and to verify the proposed heat transfer model. The IVC and SA are 310° and 265° , respectively. The calculated heat release rate of XC125 with the proposed model at 3000 rpm with 6 bar BMEP is shown in [Fig. 27](#page-9-0). Similar to the previous calculated results, the proposed model has accurate estimation results between the IVC and SA, i.e. the $\dot{Q}_{hr,IVC\rightarrow SA}$ of the proposed model is close to zero.

The cylinder pressures of XC125 at other operating conditions are also utilized to calculate $\dot{Q}_{hr,IVC\rightarrow SA}$ for evaluating the proposed

Fig. 26. Correlations between the measured and predicted T_{sp} for XC125.

Fig. 27. The heat release rate of proposed model at 3000 rpm with 6 bar BMEP for XC125.

model. The corresponding calculated results of $\dot{Q}_{hr,IVC\rightarrow SA}$ are shown in Table 9. The proposed model also can be used for other engine operating conditions with the root mean square 0.433.

The comparison of cylinder pressures for XC125 at 3000 and 6000 rpm with 6 bar BMEP (full load) are then shown in Figs. 28 and 29, respectively. Both figures also show that the proposed model has the predicted results closest to the experimental data than previous one. Since the previous model underestimates the T_{sp} (see Fig. 26), the corresponding temperature difference for calculating dQ_{ht}/dt is increased, as shown in Eq. [\(1\)](#page-1-0). Thus, it results in overestimation of cylinder pressure.

In order to compare the proposed model and previous model at all operating conditions of three engines, the difference between the R_S of the proposed model $R_{S,Proposed}$ and the R_S of the previous model $R_{S,Previous}$ is defined as

Table 9

Heat release rate $\dot{Q}_{hr,IVC\rightarrow SA}$ of XC125.

$\omega_{\rm e}$	BMEP						
	2N	3 N	4 N	5 N	6 N	7 N	
3000 rpm	-0.66	0.103	0.124	0.212	-0.279	0.354	
4000 rpm	-0.67	-0.45	0.72	0.15	0.156	0.245	
5000 rpm	-0.8	-0.54	-0.52	-0.4	0.21	0.109	
6000 rpm	-0.52	-0.6	-0.59	-0.109	-0.27	0.15	

Note: Root mean square of $\dot{Q}_{hr,IVC \rightarrow SA} = 0.433$.

Fig. 28. Comparison of the cylinder pressure at 3000 rpm with 6 bar BMEP for XC125.

Fig. 29. Comparison of the cylinder pressure at 6000 rpm with 6 bar BMEP for XC125.

$$
\Delta R_S = R_{S,Proposed} - R_{S,Pre \, vious} \tag{27}
$$

If ΔR_s is positive, the proposed model performs better than previous one, and vice versa. The detail results of R_S are shown in Appendix B. The results of ΔR_S at all operating conditions of AN125 are shown in [Table 10](#page-10-0). As can been seen from [Table 10,](#page-10-0) since the previous model is developed based on AN125, it has closer prediction results than proposed model at about 62.5% of all operating conditions with a negative mean value of -0.056 .

The results of ΔR_s at all operating conditions of CB125 are shown in [Table 11](#page-10-0). As can been seen from [Table 11](#page-10-0), the proposed model has the closer prediction results than previous one at most engine operation conditions about 79.2% with a positive mean value of 0.156, which is significantly larger than the negative mean value of -0.042 at the other engine operating conditions.

The results of ΔR_s at all operating conditions of XC125 are shown in [Table 12](#page-10-0), which shows that the proposed model has the closer prediction results than previous one at most engine operation conditions about 79.2% with a positive mean value of 0.066 . The negative mean value is -0.067 at the other engine operating conditions.

6. Conclusion

The heat transfer model proposed by previous study is not so accurate for different small-scale S.I. engines because it was developed based on single engine. In order to develop a heat transfer model suitable for more small-scale S.I. engines, a heat transfer model also using the Stanton number, which is much simpler but yet accurate enough, is proposed in this paper. The proposed model is determined as a function of compression ratio, stroke, cylinder volume, and mean piston speed. Since the spark plug temperature is required for the proposed heat transfer model, an experimental formula based on two engines is proposed, and determined as a function of intake manifold pressure, engine speed, compression ratio, and throttle opening angle. Experimental data of instantaneous heat flux and global heat transfer of two engines were used to verify the proposed heat transfer model. In addition, the experimental data of cylinder pressure and corresponding heat release rate of three engines were conducted to verify the engine model with the proposed heat transfer model. The results show that the proposed model has better prediction results than the previous one at the most engine operation conditions, thus it can be widely employed for different small-scale engines.

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Appendix A

See Table A1.

Appendix B

See Tables B1–B6.

Table 10

 ΔR_S of the cylinder pressure of AN125.

Note: Negative mean value = –0.056; positive mean value = 0.023.

Table 11

 ΔR_S of the cylinder pressure of CB125.

Note: Negative mean value = –0.042; positive mean value = 0.156.

Table 12

 ΔR_S of the cylinder pressure of XC125.

ω_e	BMEP					
	2 bar	3 bar	4 bar	5 bar	6 bar	7 bar
3000 rpm	0.046	0.101	0.116	0.142	0.048	0.119
4000 rpm	-0.046	0.023	0.128	0.064	0.013	0.079
5000 rpm	-0.057	-0.080	-0.134	-0.020	0.142	0.005
6000 rpm	0.095	0.002	0.015	0.021	0.047	0.043

Note: Negative mean value $= -0.067$; positive mean value $= 0.066$.

Table A1

Coefficients of T_{sp} .

Table B1

 R_s of cylinder pressure of the previous model for AN125.

Mean = 0.9596.

Table B2

 R_s of cylinder pressure of the proposed model for AN125.

Mean = 0.9325.

Table B3

R_s of cylinder pressure of the previous model for CB125.

Mean = 0.7925.

Table B4

 R_s of cylinder pressure of the proposed model for CB125.

Mean = 0.90725.

Table B5

 R_s of cylinder pressure of the previous model for XC125.

Mean = 0.8742.

Table B6

 R_s of cylinder pressure of the proposed model for XC125.

Mean = 0.9121.

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